tions of free hydrocarbons (oil), bitumens, argillaceous particles, bound hydrocarbons (residual oil saturation); V_i, volumes of separate sections; λ', λ_{sil}, λ_{cal}, λ_d, λ'', λ, thermal conductivities of mixtures of separate components and the effective thermal conductivy of the entire Bazhenov suite, W/(m·K); m_{cr}, volume concentration of the cracks; and c and M are parameters which depend on m_{cr}.

LITERATURE CITED

1. T. V. Dorofeeva (ed.), Oil Reservoirs of the Bazhenov Suite in Western Siberia [in Russian], Leningrad (1983).
2. G. N. Dul'nev and Yu. P. Zarichnyak, Thermal Conductivity of Mixtures and Composite Materials [in Russian], Leningrad (1974).
3. G. N. Dul'nev, Yu. P. Zarichnyak, and B. L. Muratova, Inzh.-Fiz. Zh., 31, No. 2, 278-283 (1976).
4. S. Clark, Jr. (ed.), Handbook of Physical Constants of Rocks [Russian translation], Moscow (1969).
5. Yu. L. Rastorguev, B. A. Grigor'ev, G. F. Bogatov, et al., Thermophysical Properties of Liquids [in Russian], Moscow (1976), pp. 24-26.
6. V. S. Chirkin, Thermophysical Properties of Materials Used in Nuclear Technology [in Russian], Moscow (1968).
7. V. V. Kurepin and A. F. Begunkova, Inzh.-Fiz. Zh., 29, No. 4, 241-247 (1975).

ANALYSIS OF THE CONJUGATE PROBLEM OF EVAPORATION FROM THE WALLS OF A LONG CHANNEL

V. I. Nosik

UDC 536.423.1

The conditions under which the nonequilibrium nature of the evaporation and redistribution of heat in the solid walls must be taken into account in order to calculate the heat and mass transfer in a long flat channel are determined.

The problems of evaporation from the walls of a long channel are encountered in the construction of models of physicochemical processes in porous media, as well as in the theory of heat pipes and dryers. There are two approaches to the solution of problems of this type. In the "energy" approach [1, 2] the intensity of the evaporation is determined based on the magnitudes of the heat flux flowing up to the evaporating surface, the pressure is assumed to be equal to the saturation pressure, and then the gas-dynamic problem with known blow-in intensity is solved and the distribution of the gas pressure and temperature of the evaporation surface T_0 is found. In the "kinetic" approach [3-5] the temperature of this surface is given and the pressure, gas velocity, and evaporation intensity are determined by the methods of the kinetic theory of gases. In the gas-dynamic limit (for small Knudsen numbers Kn) a relation of the Hertz–Knudsen type [6, 7], relating the flow rate with the temperature T_0 and gas pressure, is used; the pressure and flow rate are determined in the solution of the gas-dynamic problem.

Both approaches are limited, since they do not take into account the conjugate nature of the problem: the intercoupling of the heat transport in solid walls, the flow of vapor, and the kinetics of evaporation. Indeed, both the temperature of the evaporating surface (which is given in the kinetic approach) and the heat flow to it (in the energy approach) depend in the general case on the characteristics of heat transfer in the solid walls as well as on the characteristics of the flow and of the evaporation.

It is important to determine the region of applicability of these approximate approaches, clarify the necessity for using the conditions of nonequilibrium evaporation, and determine the region of "nonuniform" evaporation, when the redistribution of energy and the solution of the two-dimensional equation of heat conduction in the solid walls must be taken into account. The conjugate problem with uniform heat flow to the wall for low Reynolds' numbers of the

Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 52, No. 1, pp. 116-122, January, 1987. Original article submitted September 23, 1985.

 0022-0841/87/5201-0094$12.50

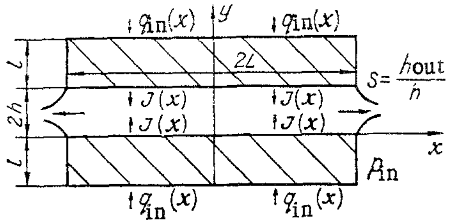

Fig. 1. Diagram of evaporation from the channel.

vapor flow Re is studied in [8]. In this work we analyze the formulations of the conjugate problem for arbitrary Re and $q_{in}(x)$.

1. We shall study evaporation from the walls of a long flat slit, at whose output a converging nozzle is placed (the gap width 2h is much smaller than the length of the nozzle 2L, $S = h_{out}/h \leq 1$). A constant pressure p_{in} is maintained on the outside. The solid walls have a finite thermal conductivity, the end surfaces are thermally insulated, and a heat flux $q_{in}(x)$ flows to the external surface (Fig. 1). The flow is assumed to be viscous and stationary.

The gas-dynamic part of the problem is described by the Navier–Stokes equations for flows in narrow channels [9] (terms $O(\delta)$, $\delta = h/L \ll 1$ are dropped), and the thermal part is described by the equation of heat conduction in a solid:

$$\partial(\rho u)/\partial x + \partial(\rho v)/\partial y = 0, \quad (1)$$

$$\rho(u\partial u/\partial x + v\partial u/\partial y) = -\partial p/\partial x + \mu\partial^2 u/\partial y^2, \quad (2)$$

$$\partial p/\partial y = 0, \quad (3)$$

$$\frac{3}{2}\rho R(u\partial T/\partial x + v\partial T/\partial y) = \lambda\partial^2 T/\partial y^2 - p(\partial u/\partial x + \partial v/\partial y) + \mu(\partial u/\partial y)^2, \quad (4)$$

$$\lambda_T(\partial^2\hat{T}/\partial x^2 + \partial^2\hat{T}/\partial y^2) = 0 \quad (5)$$

with the corresponding boundary conditions and conjugation conditions:

$$y = 0 \quad J \equiv \rho v = \alpha(p_e - p)/\sqrt{2\pi RT}; \quad (6)$$

$$y = 0 \quad T = \hat{T}, \quad -\lambda_T\partial\hat{T}/\partial y = JQ; \quad (7)$$

$$x = 0 \quad \partial\hat{T}/\partial x = 0, \quad \partial p/\partial x = 0, \quad u = 0, \quad \partial T/\partial x = 0; \quad (8)$$

$$y = -l \quad -\lambda_T\partial\hat{T}/\partial y = q_{in}(x); \quad (9)$$

$$y = h \quad \partial u/\partial y = 0, \quad v = 0, \quad \partial T/\partial y = 0; \quad (10)$$

$$x = L \quad \partial\hat{T}/\partial x = 0, \quad p = p_{out}. \quad (11)$$

Here α is a dimensionless coefficient of order unity [6, 7]; $p_e = p_* \exp(-Q/RT)$ is the saturated vapor pressure; and R is the individual gas constant.

The rate of slipping and the jump in the temperature between the gas and the wall can be neglected for low Knudsen numbers, since they do not introduce new qualitative effects, but merely give small corrections. The heat flow in the gas is also neglected compared with the energy expended on evaporation, and the additional term in the condition of the Hertz–Knudsen type is also neglected [7].

2. The gradient of the pressure arising with the acceleration of the vapor and overcoming of friction, owing to the kinetic relation (6), will generally speaking give rise to a longitudinal temperature gradient in the walls of the slit. For this reason a longitudinal heat flow, which can change the intensity of evaporation, appears. It is important to take into account the nonequilibrium nature of the evaporation in the case when the longitudinal temperature variation (associated with Δp_e) cannot be determined solely based on the pressure differential Δp.

We shall examine the conditions under which the redistribution of energy in the solid walls and the nonequilibrium nature of the evaporation must be taken into account, and we shall classify the possible simple formulations of the problem.

Integrating the equation of heat conduction (5) across the slit and taking into account the boundary conditions (7) and (9) we obtain

$$JQ \simeq q + l\lambda_T (\partial^2 \hat{T}/\partial x^2)_{av}.$$

From here we obtain an estimate of the change in the flow rate along the slit ΔJ:

$$\Delta JQ \simeq \Delta q + \frac{l\lambda_T}{L^2}\left(\Delta T_0 + \frac{\Delta q l}{\lambda_T}\right) \simeq \Delta q\,(1 + \beta^2) + \beta q_x, \tag{12}$$

where Δq is the change in the heat flux flowing in; $\beta = \ell/L \ll 1$; $q_x = \lambda_T \Delta T_0/L$ is the longitudinal heat flux, created by the differential of the temperature of the evaporating surface ΔT_0.

We introduce a quantity which is equal to the difference of the saturation and vapor pressures: $\pi \equiv p_e - p$. From (12) and (6), neglecting terms $O(\beta^2)$, we have

$$\frac{\Delta\pi}{\pi} = \frac{\Delta J}{J} \simeq \frac{\Delta q}{q} + \beta\,\frac{q_x}{q}.$$

Taking into account the fact that $\Delta\pi \equiv \Delta p_e - \Delta p$ while $\Delta p_e = p_e \bar{Q} \Delta T_0/T$, from here we obtain

$$\frac{\Delta p_e}{\Delta p} = \frac{1 + (\pi/\Delta p)\,\Delta q/q}{1 + (\pi/\Delta p_e)\,\beta q_x/q}.$$

Introducing the dimensionless parameters $H_\pi \equiv \pi/\Delta p$ and $H_p \equiv (\Delta p_e/\Delta p) q/q_x\beta$, we write

$$\frac{\Delta p_e}{\Delta p} = \frac{1 + H_\pi \Delta q/q}{1 + H_\pi/H_p}, \tag{13}$$

$$\frac{\beta q_x}{q} = \frac{1}{H_p}\,\frac{1 + H_\pi \Delta q/q}{1 + H_\pi/H_p}. \tag{14}$$

As analysis shows, this is the most convenient choice of parameters for the conditions of the given multiparameter problem (see Sec. 3).

The redistribution of energy in the solid walls is significant for determining the flow rate, when

$$\beta q_x/q \gtrsim 1. \tag{15}$$

When (15) holds, generally speaking, it is necessary to solve the two-dimensional heat conduction equation (5). In so doing, the size of the region of "nonuniform" evaporation and the function $J(x)$ will depend on q_x/q and the thickness of the solid walls ℓ.

The nonequilibrium nature of the evaporation must be taken into account when $(\Delta p_e/\Delta p - 1) \gtrsim 1$.

For clarity, the possible formulations of the problem with $\Delta q/q = O(1)$ are shown in the diagram of Fig. 2. Here the following regions are separated: $H_p \gg H_\pi$, $H_p \gg 1$ [region I, in which the subregions a) $H_\pi \ll 1$, b) $H_\pi \sim 1$, and c) $H_\pi \gg 1$ are labeled]; $H_p \sim H_\pi \gg 1$ (region II); $H_p \ll H_\pi$, $H_\pi \gg 1$ (region III); $H_p \ll H_\pi$, $H_\pi \sim 1$ (region IV); $H_p \sim H_\pi \sim 1$ (region V); $H_p \gg H_\pi$, $H_p \sim 1$ (region (VI). In regions I and VI the condition of equilibrium evaporation $p = p_e$ can be used to determine the flow rate; in regions II-VI the redistribution of energy in the solid walls is important.

We shall examine each region in greater detail.

In region I the transverse heat flow, according to (14), is weak, as a result of which the flow rate can be found without solving the equation of heat conduction in the solid: $J(x) = q_{in}(x)/Q$. The gas-dynamic problem (1)-(4) with the known intensity of blow-in is solved independently of the thermal problem, as a result of which $p(x)$ is determined. In the region Ia, according to (13), $p_e(x) = p(x)$ and the temperature of the wall is calculated without the use of the kinetic relation (6). In the regions Ib and Ic the change in the temperature cannot be found without (6), and in addition in the region Ic p = const (this follows from the definition of H_π). The approach of [1, 2] is valid in the region Ia.

In the region II the pressure is also constant, but in order to calculate the flow rate the two-dimensional heat conduction equation (5) with the condition (6) with p = const must be solved. In the region III, according to (13) and the definition of H_π, p_e = const and p = const. Therefore the evaporation surface T_0 and, according to (6), the flow rate J are constant. The flow rate is determined from the relation

$$J = (LQ)^{-1} \int_0^L q_{in}(x)\, dx.$$

The kinetic approach is valid in region IV (like in III, where it reduces to the degenerate case). Here T_0 = const for arbitrary $q_{in}(x)$, and the gas-dynamic problem (1)-(4) with conditions of the Hertz–Kundsen type (6) with p_e = const must be solved to in p(x) and J(x).

In the region V the problem in its general formulation (1)-(11) must be solved. The temperature of the evaporation surface changes along x by an amount $O(Q^{-1}\pi/p) \ll 1$ and the pressure changes by an amount $\Delta p/p \sim \pi/p \ll 1$, but in order to determine the flow rate the heat conduction equation must be solved using (6).

In the region VI the kinetic relation reduces to $p_e(x) = p(x)$, and the solution is independent of the form of the kinetic model. To determine the flow rate, pressure, and temperature $T_0(x)$, however, the heat (5) and gas-dynamic (1)-(4) problems must be solved simultaneously under the condition $p(x) = p_e(x)$. The pressure drops need not be small.

In the limits H_π, $H_p \to 0$, according to (14), $q_x \to \infty$, i.e., strong nonuniformity is observed and the estimates presented above are not valid.

3. To determine the parameters H_π and H_p we shall estimate the characteristic values of the velocity and the pressure and its differentials.

The pressure at the outlet from the gap can be evaluated from the problem of nonviscous adiabatic efflux of gas from a nozzle [10]. Choosing for the stagnation parameters of the flow the saturation pressure and the temperature at the inlet into the nozzle, we express the flow rate in terms of these parameters:

$$\rho u_{av}|_{x=L} = AS\delta p/\sqrt{2\pi RT}\; f(p/p_{in}),\quad A = \sqrt{\varkappa}\left(\frac{\varkappa+1}{2}\right)^{-\frac{\varkappa+1}{2(\varkappa-1)}};$$

$$f\left(\frac{p}{p_{in}}\right) = \begin{cases} 1, & p \geqslant p_{in} = p_{cr}\left(\frac{\varkappa+1}{2}\right)^{\frac{\varkappa}{\varkappa-1}}, \\ \sqrt{\frac{2}{\varkappa-1}}\left(\frac{\varkappa+1}{2}\right)^{\frac{\varkappa+1}{2(\varkappa-1)}} \times \left(\frac{p}{p_{in}}\right)^{-\frac{\varkappa+1}{2\varkappa}}\sqrt{\left(\frac{p}{p_{in}}\right)^{\frac{\varkappa-1}{\varkappa}} - 1}, & p_{in} < p < p_{cr}, \\ 0, & p \leqslant p_{in}. \end{cases}$$

The flow rate is determined by the energy introduced, $\rho u_{av} = \int_0^L q_{in}(x)\, dx/hQ$, whence

$$\begin{aligned} p_{out} &\sim \max\{p_{in},\ (q/Q)(S\delta)^{-1}\sqrt{RT}\}, \\ M_{out} &\sim S\min\{1,\ (q/Q)(S\delta)^{-1}\sqrt{RT}/p_{in}\}. \end{aligned} \tag{16}$$

From the equation of continuity (1) we obtain $v \sim u\delta$; thus the evaporation in the channel will be "weak," $M_{evap} \simeq \pi/p \lesssim S\delta \ll 1$ (excluding, possibly, the end regions) irrespective of the value of the heat inflow. However, the longitudinal velocity u for low values of p_{in} and large values of q_{in} can be significant compared with the velocity of sound.

Estimating the pressure differential along the gap, from the momentum equation (2) we obtain:

$$\begin{aligned} \frac{\Delta p}{p} &\simeq \frac{\rho u \Delta u}{\rho RT} + \frac{\mu \Delta u L}{\rho RT h^2} \simeq O(M^2)(1 + O(1/\mathrm{Re})), \\ \mathrm{Re} &= \frac{\rho u h^2}{\mu L} \simeq \frac{\rho v h}{\mu} \simeq \frac{qh}{Q\mu}, \end{aligned} \tag{17}$$

where Re is the Reynolds number for the flows in the long channels (flow of the Hill-Shaw type [11]), which is determined in this case by the width of the slit, the heat inflow, the heat of evaporation, and the viscosity of the gas. The pressure drops may not be small for

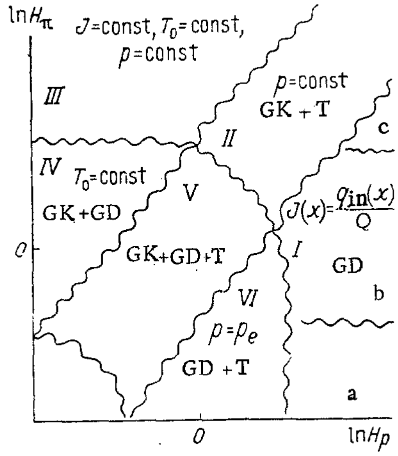

Fig. 2. Diagram of possible formulations of the problem in the leading order approximation: GD) the gas-dynamic problem (1)-(4); T) thermal problem (5); GK) conditions for nonequilibrium evaporation (6).

$M = O(1)$ or even for $M^2/Re \gtrsim 1$. At the same time the pressure drop across the slit $\Delta_y p/p = O(\delta) \ll 1$ [9].

Using these estimates for $\Delta p/p$ and π/p and also the relations $\Delta p_e \simeq \bar{Q}\Delta T_0/T$ and $q_x \simeq \lambda_T \Delta T_0/L$, from the definitions of H_π and H_p we obtain:

$$H_\pi = \delta M^{-1} \min\{Re,\ 1\},\quad H_p = M^{-2}(\bar{Q}qL/\lambda_T T)\,\beta^{-1} \min\{Re,\ 1\},$$

where M is given by the relation (16) and Re is given by (17).

Thus the values of the parameters H_π and H_p, determining the evaporation regimes, depend on the flow regime: the numbers M and Re, the geometric parameters $\delta = h/L$ and $\beta = \ell/L$, and the complex $\bar{Q}qL/\lambda_T T$.

The condition for redistribution of energy in the solid (15) assumes the form

$$\beta\,\frac{\lambda_T T}{\bar{Q}Lq}\,\frac{\max\{M^2,\ M^2/Re,\ M\delta\Delta q/q\}}{1 + O(\beta M\delta\lambda_T T/\bar{Q}qL)} \gtrsim 1, \tag{18}$$

while the necessary condition for using the conditions of nonequilibrium evaporation is

$$\beta M\delta\,\frac{\lambda_T T}{\bar{Q}qL} + \frac{\Delta q}{q}\,\frac{\delta}{M}\,\min\{Re,\ 1\} \gtrsim 1,$$

We rewrite the last relation for q_{in} = const in the form

$$1 \gtrsim \frac{1}{\beta}\,\frac{1}{M\delta}\,\frac{\bar{Q}qL}{\lambda_T T} \sim \frac{1}{\beta}\,\bar{Q}\,\frac{qLR\mu}{\lambda_T TR\mu}\,\frac{p}{Ja} \sim \frac{1}{\beta}\left(\frac{Q}{RT}\right)^2 \frac{1}{\delta\,Kn}\left(\frac{\lambda}{\lambda_T}\right), \tag{19}$$

where Knudsen's number $Kn = a\mu/ph$ is determined based on the pressure at the inlet into the nozzle. Thus the kinetic relation (6) for the intensity of evaporation in the problem under study must be used for materials with high thermal conductivity in the solid phase with not too small numbers Kn and quite short slits.

For $\Delta q/q \sim 1$, in addition to (19), the inequality $M \lesssim \delta \min\{Re, 1\}$, i.e., the condition that the flow be slow, must hold. When these conditions are satisfied the kinetic approach is valid [3].

We shall now examine in greater detail the condition under which the flow of heat in the solid can be neglected (18) (the condition that evaporation is nonuniform). For sufficiently small values of λ_T the evaporation will always be uniform, and in addition for $H_\pi \ll 1$ (quite large pressure drops along the slit) the energy approach is valid.

As M decreases (or Re increases) the nonuniformity decreases (owing to the decrease in the pressure drop and therefore in $\Delta_x T$). The nonuniformity also decreases as the slit length increases, T decreases, and the heat of evaporation increases.

We note that the arguments presented above refer to the case of "strong" nonuniformity, when the flow rate changes in the entire region. The local disturbance of the flow rate in the end region of the gap, where the gradients are largest, occurs under less rigid conditions [8].

NOTATION

h, width of the channel; L, length of the channel; $\delta = h/L$, relative width of the channel; ℓ, thickness of the walls; $\beta = \ell/L$, relative thickness of the walls; p, gas pressure; p_e, saturation pressure; $\pi = p_e - p$, pressure drop; T, temperature; T_0, temperature of the evaporation surface; Q, heat of evaporation; $\bar{Q} = Q/RT$, dimensionless heat of evaporation; μ, viscosity of the gas; λ, thermal conductivity of the gas; λ_T, thermal conductivity of the solid phase; q, heat flux; J, mass flow rate; a, velocity of sound; M, Mach's number; Re, Reynold's number; Kn, Knudsen number; and H_π and H_p, dimensionless complexes.

LITERATURE CITED

1. P. A. Novikov, G. L. Malenko, L. Ya. Lyubin, and V. I. Balakhonova, Inzh.-Fiz. Zh., 22, No. 5, 811-817 (1973).
2. P. A. Novikov, G. L. Malenko, and L. Ya. Lyubin, Inzh.-Fiz. Zh., 26, No. 1, 58-63 (1974).
3. G. E. Gorelik, N. V. Pavlyukevich, V. V. Levdanskii, et al., Physical Kinetics and Transport Processes Accompanying Phase Transformations [in Russian], Minsk (1980).
4. V. G. Leitsina, N. V. Pavlyukevich, T. L. Perel'man, and G. N. Rudin, Inzh.-Fiz. Zh., 29, No. 5, 295-300 (1975).
5. L. N. Shulepov, Inzh.-Fiz. Zh., 35, No. 1, 82-86 (1978).
6. S. Knake and I. N. Stranskii, Usp. Fiz. Nauk, 68, No. 2, 261-305 (1959).
7. N. K. Makashev, Uch. Zap. TsAGI, 5, No. 3, 49-62 (1974).
8. V. I. Nosik, Proceedings of the 9th Conference of Young Scientists of FALT, Moscow (1984), pp. 4-9.
9. J. C. Williams, AIAA J., 1, No. 1, 186-195 (1963).
10. L. G. Loitsyanskii, Mechanics of Liquids and Gases [in Russian], Moscow (1978).
11. G. Shlichting, Boundary Layer Theory, McGraw-Hill (1960).

LIMITING SOLUTION OF A DIFFUSIONAL PROBLEM IN PRISMATIC TUBES

A. I. Moshinskii

UDC 532.72

The limiting solution of the convective-diffusion equation is investigated in channels close to the tube "axis," i.e., the line at which the liquid flow rate takes on a maximum value.

The solution of heat- and mass-transfer problems in prismatic tubes, even when the tubes are linear, entails well-known difficulties when using the methods of mathematical physics [1]. In connection with this, approximate methods are widely used: numerical methods [2], variational and projectional methods [3], methods based on introducing an effective (Taylor) diffusion coefficient [4], and various modifications and improvements of these [5-7].

In the present work, small-perturbation theory is used to investigate a characteristic solution of the problem of impurity propagation in prismatic tubes at large Peclet numbers. The behavior of the impurity concentration around the tube axis is of interest here. The liquid is assumed to be Newtonian and the liquid flow to be laminar, although the individual assumptions of the theory and calculations may simply be extended to more complex cases.

I. Plane Channel

Suppose that the liquid is of sufficiently high viscosity that the liquid flow is stabilized over time and, at the same time, the diffusional process is unstable. The impurity-diffusion equation in this case takes the form

Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 52, No. 1, pp. 122-131, January, 1987. Original article submitted October 8, 1985.

0022-0841/87/5201-0099$12.50